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ACTIVATION OF NUCLEATION CAVITIES ON A HEATING SURFACE WITH TEMPERATURE GRADIENT IN SUPERHEATED LIQUID

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Abstract—The nuclei of vapour bubbles in pool boiling are usually assumed to be spheres of radius R_n, the value of which depends upon the superheat of liquid. It is shown in this paper that the vapour nucleus is a sphere only in the case of uniform superheat. If there is a temperature gradient the shape of the active bubble nucleus is flattened. As a consequence the liquid superheat at the wall needed for activation is greater than in the case of uniform superheat.

NOMENCLATURE

b, height of the nucleus;
k, a parameter, equation (27);
p, pressure;
p'_s, $= (\mathrm{d}p/\mathrm{d}T)_{T=T_s}$;
r, co-ordinate;
R_c, radius of the cavity;
R_n, radius of the vapour nucleus;
R_1, R_2, main radii of curvature of the vapour nucleus;
T, temperature of the liquid;
T_s, absolute saturation temperature;
ΔT, superheat at the heated surface;
∇T, temperature gradient at the heated surface;
y, co-ordinate;
β, contact angle;
γ, a parameter, equation (14);
δ, boundary-layer thickness;
θ, characteristic superheat of the nucleus;
ρ', ρ'', mass densities of the liquid and of the vapour, respectively;
σ, surface tension;
φ, angle between the surface of the nucleus and the heated surface, at the heated surface.

IN POOL BOILING the working period of an active site on the heated surface consists of two parts. In the first the vapour bubble is formed from the initial nucleus. After the departure of the grown-up bubble the site is not active for a while, and this period is called the waiting period. To explain this phenomenon Hsu [1] has proposed a model in which a vapour nucleus of radius R_n, seated on the cavity of radius R_c, begins to be active (that is, begins to grow) at the moment when the temperature of the surrounding superheated liquid exceeds the temperature of the characteristic superheat θ due to the radius of the nucleus R_n,

$$\theta = \frac{2\sigma}{p'_s R_n} \cdot \frac{\rho'}{\rho' - \rho''}. \tag{1}$$

After the departure of the bubble the colder liquid approaches the wall. During the waiting period the superheat of the liquid is initially below the value θ at the place $y = b$, which the vapour nucleus reaches (see Fig. 1). Therefore the bubble does not grow, unless the superheat at the place $y = b$ exceeds the prescribed value θ.

Hsu's hypothesis is a good explanation of the existence of the waiting period, but has no physical basis. One must ask why the activation of a nucleus is governed by the superheat at the place $y = b$, and why not at $y = 0$, that is on the heated surface.

Note that the radius of the active nucleus is evaluated from the formula (1), which is strictly valid for uniform superheat of the liquid only. Namely, the expression (1) is the solution of the Laplace equation

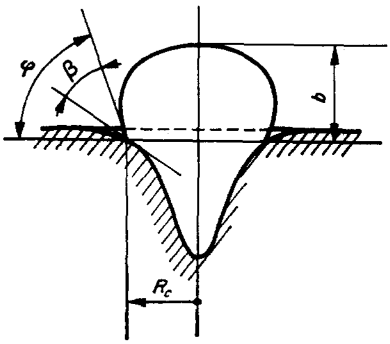

FIG. 1.

$$\frac{\rho' - \rho''}{\rho'} \cdot \frac{\Delta p}{\sigma} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2}$$

in which we put

$$\Delta p \approx p_s'(T - T_s) \tag{3}$$

For $T =$ const. and $(T - T_s) =$ const. $= \theta$, the nucleus is spherical, that is $R_1 = R_2 = R_n$, and equation (1) is obtained. In the conditions of pool boiling, however, the temperature of the liquid is a function of space and time, as pointed out by Hsu [1]. Thence the temperature difference $(T - T_s)$ varies with the co-ordinate y and influences the radii of the active nucleus, which depend therefore not only upon the superheat at the wall (which may be assumed constant), but also upon the temperature distribution in the vicinity of the active site (the cavity), and consequently upon the momentary heat flux, or temperature gradient, at the heated surface. The greater the temperature gradient at the wall, the greater must be the radius of the cavity to produce bubbles, even at constant superheat of the wall.

To analyse the phenomenon we put

$$\Delta p \approx p_s' \cdot (T - T_s) = p_s' \cdot \Delta T \vartheta(y) \tag{4}$$

where

$$\vartheta(y) = \frac{T - T_s}{\Delta T}, \quad \vartheta(0) = 1 \tag{5}$$

Using the expressions for the main radii of curvature:

$$R_1 = -\frac{1}{y''}(1 + y'^2)^{3/2},$$

$$R_2 = -\frac{r}{y'}(1 + y'^2)^{1/2} \tag{6}$$

where $y' = dy/dr$, $y'' = d^2y/dr^2$, we obtain from equation (2)

$$-\frac{\rho' - \rho''}{\rho'} \cdot \frac{p_s'\Delta T}{\sigma} \vartheta(y) = \frac{y''}{(1 + y'^2)^{3/2}} + \frac{y'}{r(1 + y'^2)^{1/2}}. \tag{7}$$

The boundary conditions, which follow from the sketch in Fig. 2, are

$$y(R_c) = 0, \quad y'(R_c) = tg\varphi, \quad y'(0) = 0 \tag{8}$$

We confine ourselves to the analysis of the case $\varphi = \pi/2$. The angle φ, as it can be seen from Fig. 1, is usually different from the contact angle β; it has to do with the microgeometry of the heated surface.

In the case of $\varphi = \pi/2$ and uniform superheat of the liquid ($\vartheta = 1$) the nucleus forms a hemisphere of radius $R_n = R_c$, and height $b = R_c$. If $\vartheta(y) \leqslant 1$ the shape of the nucleus resembles a flattened spheroid. For a real spheroid we would obtain

$$y = b\left(1 - \frac{r^2}{R_c^2}\right)^{1/2} \tag{9}$$

and the ratio of the radii of curvature,

$$\frac{R_1}{R_2} = \frac{y'(1 + y'^2)}{ry''} = 1 - \left(1 - \frac{b^2}{R_2^c}\right) \cdot \frac{r^2}{R_c^2} \tag{10}$$

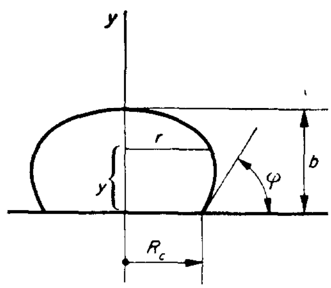

FIG. 2.

varies from $R_1/R_2 = 1$ at the top ($r = 0$) to $R_1/R_2 = b^2/R_c^2$ at the base ($r = R_c$). The arithmetical mean of the discussed ratio is therefore

$$\left(\frac{R_1}{R_2}\right)_m = \tfrac{1}{2}\left(1 + \frac{b^2}{R_c^2}\right) \tag{11}$$

We will solve the problem approximately using instead of equation (2) the simplified equation

$$\frac{\rho' - \rho''}{\rho'} \cdot \frac{\Delta p}{\sigma} = \frac{1}{R_1}\left[1 + \left(\frac{R_1}{R_2}\right)_m\right] \tag{12}$$

or

$$-\gamma\vartheta(y) = \frac{R_c y''}{(1 + y'^2)^{3/2}} \tag{13}$$

where

$$\gamma = \frac{\rho' - \rho''}{\rho'} \cdot \frac{R_c p_s' \Delta T}{\sigma} \cdot \frac{1}{1 + (R_1/R_2)_m} \tag{14}$$

Putting $y' = u$, $y' = u\,\mathrm{d}u/\mathrm{d}y$, we obtain by integration

$$\frac{\gamma}{R_c}\int_0^y \vartheta\,\mathrm{d}y + \text{const.} = \frac{1}{(1 + y'^2)^{1/2}} \tag{15}$$

where const. = 0, which follows from the boundary conditions (8) for $\varphi = \pi/2$. Thus

$$y' = \frac{\mathrm{d}y}{\mathrm{d}r} = -\sqrt{\left[\left(\frac{\gamma}{R_c}\int_0^y \vartheta\,\mathrm{d}y\right)^{-2} - 1\right]} \tag{16}$$

and

$$r = R_c - \int_0^y \frac{\mathrm{d}y}{\sqrt{[(\gamma/R_c \int_0^y \vartheta\,\mathrm{d}y)^{-2} - 1]}} \tag{17}$$

where conditions (8) are already taken into account. If $r = 0$, that is $y = b$, it follows $y' = 0$, whence

$$R_c = \int_0^b \frac{\mathrm{d}y}{\sqrt{[(\gamma/R_c \int_0^y \vartheta\mathrm{d}y)^{-2} - 1]}} \tag{18}$$

$$\frac{\gamma}{R_c}\int_0^b \vartheta\,\mathrm{d}y = 1 \tag{19}$$

From these equations the value of b should be eliminated and as a result we obtain the relationship $R_c = f(\gamma/R_c)$. Note that for $\vartheta = 1$ the result is $\gamma = 1$, which leads to the formula

$$\Delta T = \frac{2\sigma}{p_s' R_c} \cdot \frac{\rho'}{\rho' - \rho''},$$

where for $\varphi = \pi/2$ it is $R_n = R_c = b$.

To discuss the case $\vartheta(y) \leqslant 1$ we must assume a defined temperature distribution. To avoid mathematical difficulties we analyse the simplest case, as shown in Fig. 3. Thus

$$\left.\begin{aligned} \vartheta &= 1 - \frac{y}{\delta} \quad \text{for } y \leqslant \delta; \\ \vartheta &= 0 \qquad\quad \text{for } y > \delta. \end{aligned}\right\} \tag{20}$$

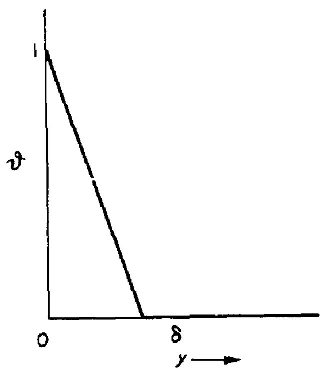

FIG. 3.

The following considerations are valid for $b \leqslant \delta$ only. Therefore

$$\int_0^b \vartheta\,\mathrm{d}y = y - \frac{y^2}{2\delta} \tag{21}$$

and

$$\frac{\gamma}{R_c}\left(b - \frac{b^2}{2\delta}\right) = 1 \tag{22}$$

We use a new variable t, satisfying the equation

$$\frac{\gamma}{R_c}\left(y - \frac{y^2}{2\delta}\right) = 1 - t^2 = \frac{\gamma}{R_c}\int_0^y \vartheta\,\mathrm{d}y. \tag{23}$$

Hence

$$y = \delta\left\{1 - \sqrt{\left[1 - \frac{2R_c}{\delta\gamma}(1 - t^2)\right]}\right\}$$

and

$$dy = -\frac{2R_c}{\gamma} \cdot \frac{tdt}{\sqrt{[1 - (2R_c/\delta\gamma)(1 - t^2)]}} \quad (24)$$

Introducing (23) and (24) into equation (18) we get finally after some rearrangement

$$\gamma = 2\int_0^1 \frac{(1 - t^2)\,dt}{\sqrt{[2 - t^2]}\sqrt{[1 - (2R_c/\delta\gamma)(1 - t^2)]}}. \quad (25)$$

Using furthermore the Legendre substitution

$$t = \sqrt{2}\,.\cos\psi \quad (26)$$

and introducing the quantity

$$k = \left(\frac{\delta\gamma}{R_c} - 1\right)^{-1/2} \quad (27)$$

we obtain from equation (25)

$$\gamma = 2\sqrt{[1 + k^2]}\int_{\pi/4}^{\pi/2} \frac{(2\sin^2\psi - 1)\,d\psi}{\sqrt{[1 - k^2\sin^2\psi]}} \quad (28)$$

This integral may be expressed in terms of the elliptic integrals of first and second kinds

$$\left.\begin{aligned} F(k, \psi) &= \int_0^\psi \frac{d\psi}{\sqrt{[1 - k^2\sin^2\psi]}} \\ E(k, \psi) &= \int_0^\psi \sqrt{[1 - k^2\sin^2\psi]}\,d\psi, \end{aligned}\right\} \quad (29)$$

The result is

$$\gamma = 2\sqrt{[1 + k^2]}\left\{\left(\frac{2}{k^2} - 1\right)\cdot\left[F\left(k, \frac{\pi}{2}\right) - F\left(k, \frac{\pi}{4}\right)\right] - \frac{2}{k^2}\left[E\left(k, \frac{\pi}{2}\right) - E\left(k, \frac{\pi}{4}\right)\right]\right\} \quad (30)$$

The value of $(R_1/R_2)_m$, which appears in equation (14), may be also expressed in terms of the quantity k, given by equation (27). Namely, it follows from equation (22) in connection with (27) that

$$b = \delta\left\{1 - \sqrt{\left[1 - \frac{2R_c}{\delta\gamma}\right]}\right\} = \delta\left\{1 - \sqrt{\left[\frac{1 - k^2}{1 + k^2}\right]}\right\} \quad (31)$$

whence

$$1 + \left(\frac{R_1}{R_2}\right)_m = 1 + \tfrac{1}{2} + \left\{1 + \left(\frac{\delta}{R_c}\right)^2 \cdot \left(1 - \sqrt{\left[\frac{1 - k^2}{1 + k^2}\right]}\right)^2\right\} \quad (32)$$

It can be seen that for $k = 0$ it is $b = 0$, and substitution $k = 1$ yields $b = \delta$. Thus

$$0 \leqslant k \leqslant 1.$$

The relationship (30) may be transformed by use of equation (27); we obtain

$$\frac{R_c}{\delta} = \frac{2}{\sqrt{[1 + k^2]}}\cdot\left\{(2 - k^2)\left[F\left(k, \frac{\pi}{2}\right) - F\left(k, \frac{\pi}{4}\right)\right] - 2\,.\left[E\left(k, \frac{\pi}{2}\right) - E\left(k, \frac{\pi}{4}\right)\right]\right\} \quad (33)$$

This relationship is shown in Fig. 4. Using equations (14), (27), (32) and (33) we may find the function

$$\left(\frac{\rho' - \rho''}{\rho'}\cdot\frac{R_c p_s' \Delta T}{2\sigma}\right) \text{ versus } \frac{R_c}{\delta};$$

this is shown in Fig. 5, and the relationship b/R_c vs. R_c/δ as well.

Now, the ratio R_c/δ is the dimensionless temperature gradient ∇T, since

$$\nabla T = \frac{\Delta T}{\delta} \quad (34)$$

whence

$$\frac{R_c}{\delta} = \frac{R_c \nabla T}{\Delta T} \quad (35)$$

If the liquid is uniformly superheated, it is $\delta = \infty$ and the nucleus is activated at

$$\Delta T = \Delta T_\infty.$$

If for instance $R_c/\delta = 3$ we obtain from the graph in Fig. 5

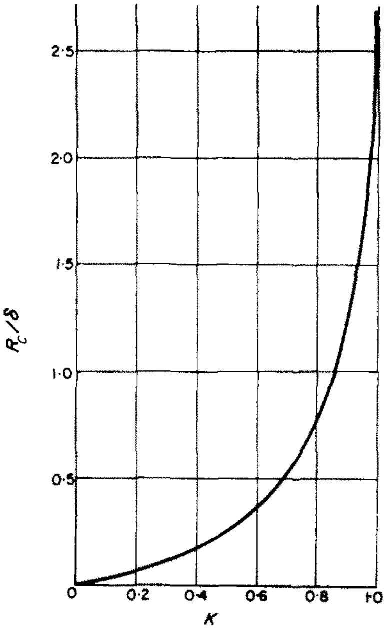

FIG. 4.

$$\frac{\rho' - \rho''}{\rho'} \cdot \frac{R_c p_s' \Delta T}{2\sigma} = 4{\cdot}66$$

wherefore the nucleus may be activated at $\Delta T = 4{\cdot}66 \Delta T_\infty$. The supposed mechanism of the waiting period is therefore as follows.

At the moment of bubble departure from the active cavity the temperature gradient may be small. The starting bubble leaves a vapour nucleus of radius R_c (for $\varphi = \pi/2$). As the result of bubble motion the colder liquid comes nearer to the wall, so that the temperature gradient grows, and the nucleus decreases due to condensation, thereby becoming flatter. Now, since the wall is held at the same temperature, the liquid grows warmer, and the temperature gradient decreases thus allowing the activation of the nucleus, consisting in spontaneous growth of it.

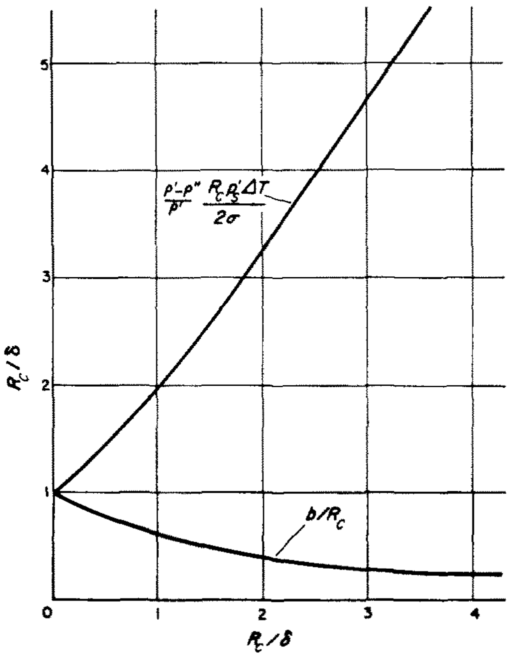

FIG. 5.

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Résumé—On suppose habituellement que les germes des bulles de vapeur dans l'ébullation en réservoir, sont des sphères de Rayon R_n, dont la valeur dépend de la surchauffe du liquide. On montre dans cet article que le germe de vapeur n'est une sphère que dans le cas d'une surchauffe uniforme. S'il y a un gradient de température. le germe d'une bulle active prend une forme aplatie. En conséquence, la surchauffe du liquide à la paroi nécessaire pour l'activation est plus grande que dans le cas d'une surchauffe uniforme.

Zusammenfassung—Die Keime von Dampfblasen beim Sieden in freier Konvektion werden gewöhnlich als Kugeln vom Radius R_n angenommen, deren Grösse von der Überhitzung der Flüssigkeit abhängt. In der vorliegenden Arbeit wird gezeigt, dass der Dampfkeim nur im Fall gleichmässiger Überhitzung eine Kugel ist; bei einem Temperaturgradienten ist der aktive Blasenkeim abgeflacht. Als Folge davon muss and er Wand die notwendige Flüssigkeitsüberhitzung zur Aktivierung grösser sein als bei gleichmässiger Überhitzung.

Аннотация—Обычно считается, что ядра пузырьков пара при кипении в большом объеме представляют собой сферы радиуса R_n величина которых зависит от степени перегрева жидкости. В статье показано, что ядро пузырька пара имеет сферическую форму только в случае равномерного перегрева. При наличии температурного градиента активные пузырьки сплющиваются. Поэтому для активации требуется большая степень перегрева жидкости на стенке по сравнению со случаем равномерного перегрева.